

**Nomenclature**

$a_{c,i}$	Input/output coefficients of a process or activity $i$	$e_c$	Right hand side coefficients or parameters of the constraints
$bc_{j,i}$	Environmental burden coefficients	$E_k$	Total Environmental impact
$b_{j,l}$	Marginal burden allocated to the co-products	$f_i$	Coefficients in the economic objective function
$b_{j,m}$	Marginal burden allocated to the material availability	$F$	Economic objective function
$b_{j,u}$	Marginal burden allocated to the capacity	$H_i$	Heat requirement in the system
$b_{j,v}$	Marginal burden allocated to the heat requirement	$P_i$	Product output
$B_j$	Total Environmental burden	$Q_i$	Heat availability
$C_u$	Capacity of a process of an operation unit	$R_m$	Primary or raw material availability
$D_l$	Market demand on the output of the products	$S_m$	Supply of primary or raw material
$ec_{k,i}$	Environmental impact coefficients	$x_i$	Output from a process or activity (operation level)
$ei_{k,m}$	Marginal impact allocated to the material availability	$\lambda_{i,c}$	Marginal or dual value of the $c$ th constraint; equal to the marginal allocated burden
$ei_{k,u}$	Marginal impact allocated to the capacity		
$ei_{k,v}$	Marginal impact allocated to products	$\mu_{k,c}$	Marginal or dual value of the $c$ th constraint; equal to the marginal allocated impact
$ei_{k,v}$	Marginal impact allocated to the heat requirement		

**Forthcoming (No. 1, 2000): Part 2****Allocation of Environmental Burdens in Co-product Systems: Process and Product-related Burdens (Part 2)**

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**Abstract.** ISO 14041 requires that allocation by physical causality must reflect the quantitative changes in product outputs or functions and will not necessarily be in proportion to simple physical measure such as mass. This paper examines the instances where physical causality can be represented by mass. However, it also goes further than ISO to demonstrate that the type of causality in the system is not necessarily always the same and can change depending on the way the system is operated. Whole system modelling and the marginal allocation approach are used to identify the correct type of causality for different operating states of the system and the corresponding changes in the environmental burdens. This is generally not possible with the other allocation methods, also examined in this paper. Both process- and product-related burdens are considered and the approach is illustrated by a reference to an existing system producing five boron co-products.

**Keywords:** Allocation; boron; environmental impacts; LCA; Life Cycle Assessment; linear programming; marginal values; system analysis

ture, allocation should be avoided where possible by system disaggregation or by expanding system boundaries. If that is not feasible, then the environmental burdens should be partitioned among different functions of the system in a way which reflects the underlying physical causality. This implies that the allocated burdens must follow the quantitative changes in product outputs or functions, which will not necessarily be in proportion to simple physical measure such as mass.

This paper focuses on allocation by physical causality, the second step in the ISO 14041 hierarchy. However, it goes further than ISO to demonstrate that the type of physical causality and, hence, the allocated burdens in a system are not necessarily fixed but can change depending on the way the system is operated. Marginal allocation and whole system modelling (AZAPAGIC and CLIFT, 1998, 1999a) are used to identify the "active" or relevant physical causality and allocation parameters in the system. These considerations are illustrated by a "cradle to gate" study of an industrial multiple-function system producing boron co-products (AZAPAGIC and CLIFT, 1999b).

**Introduction**

ISO 14041 (1998) set guidelines for dealing with allocation in multiple-function systems. According to this proce-